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21/20

Math 1050 Optimization Project
Maximize and Minimize

Nice Work!

Introduction

Often, in manufacturing, decisions must be made that involve optimization: minimizing costs, minimizing materials waste, maximizing profit, etc. In this project, you are going to examine the process of maximizing the volume of a container that is made from a given amount of material and minimizing the cost of manufacturing based on a set volume.

Part I: Maximizing the volume of an closed box

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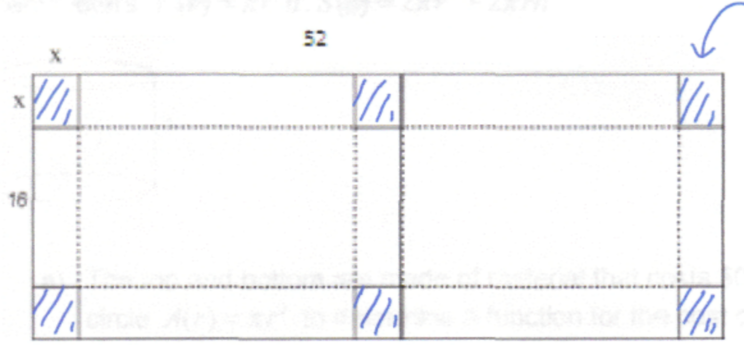
Consider making a box with a lid which is made from a 16" by 52" rectangular piece of cardboard. This is accomplished by cutting equal sized squares from each corner and one square out of the middle top and middle bottom. Then the box is made by folding up the sides. Let x be the side of the squares that are cut out.

/// denotes material that will be removed



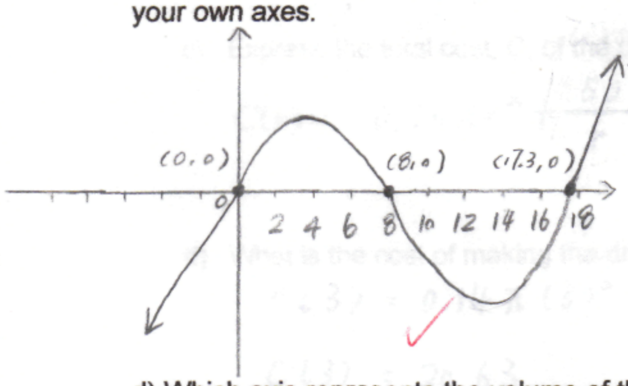
(a) $V = L \cdot W \cdot H$

a) Write the function V which gives the volume of the box as a function of x. $V(x) = [(52-3x) \div 2](16-2x)(x)$

b) What is the 'real world' domain of V? (b) $2x < 16$

$x < 8$ $\therefore$ 'Real World' domain $= (0, 8)$

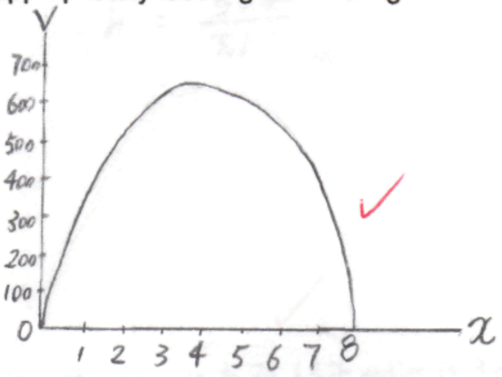
c) Graph $y = V(x)$ on an appropriate viewing window to get a picture which shows the turning point and x-intercepts of the function on its real-world domain. Sketch what you see by appropriately labeling and scaling your own axes.



Zeros	0	17.3	8
Multiplicity	1	1	1
Touch / Cross	Cross	Cross	Cross

Power function $= 6x^3$

$\therefore$ Behaviour $= \downarrow \uparrow$



d) Which axis represents the volume of the box and which axis represents the side of each cut-out square?

y-axis represents the volume of the box and x-axis represents the side of each cut-out square.

e) If the cut-out squares have a side of 3 in, then what is the volume of the box?

$V(3) = [(52-3(3)) \div 2][16-2(3)](3)$

$= (21.5)(10)(3)$

$= 645$ cubic inches

f) If the volume of the box is 400 cubic inches, then what is the side of the cut-out square?

-1 If the volume of the box is 400 cubic inches, then the side of the cut-out square is ≈ 1.2206 inches. or ≈ 6.05 in

g) What is the maximum volume of the box?

The maximum volume of the box is $653.81\ in^3$ (rounded to two decimal places.)

h) What is the side of the cut-out square for the box of maximum volume?

For the box of maximum volume, the side of the cut-out square is 3.44 inches. (rounded to two decimal places)

Part II: Finding the minimum cost of a steel drum

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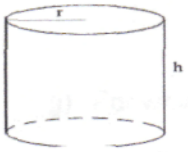
Background: A drum in the shape of a right circular cylinder is required to have a volume of 500 cubic centimeters. $V(r) = \pi r^2 h$. $S(r) = 2\pi r^2 + 2\pi rh$



a) The top and bottom are made of material that costs $0.07 per square centimeter. Use the area of a circle $A(r) = \pi r^2$ to determine a function for the cost of the top and bottom of the drum.

$0.07\ (2\pi r^2)$

$= 0.14\pi r^2$

b) The sides are made of material that costs $0.05 cents per square centimeter. Use the surface area of the sides of a right circular cylinder $Surface\ Area\ of\ Sides = 2\pi rh$ to find a function for the cost of the sides in terms of the radius, r. Hint, you need to use the volume formula and solve for the height in terms of the radius.

$0.05\ (2\pi rh)$

$= 0.05\ (2\pi r \cdot \frac{500}{\pi r^2})$

$= 0.05\ (\frac{1000}{r}) = \frac{50}{r}$

* $V = \pi r^2 h$

$500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$ *

c) Express the total cost, C, of the drum as a function of the radius.

$C(r) = 0.14\pi r^2 + \frac{50}{r}$

d) What is the cost of making the drum if the radius is 3 cm?

$C(3) = 0.14\pi (3)^2 + \frac{50}{(3)}$

$C(3) = 20.63$

→ So the cost of making the drum is \$20.63 if radius is 3 cm.

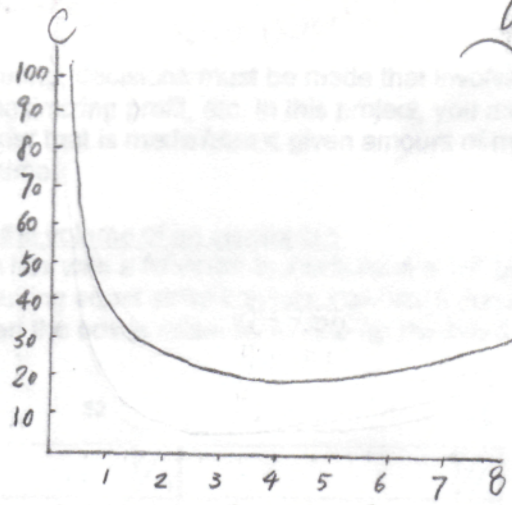
e) What is the cost of making the drum if the radius is 14 cm?

$C(14) = 0.14\pi (14)^2 + \frac{50}{(14)}$

$= 89.78$

→ So the cost of making the drum is \$89.78 if the radius is 14 cm.

f) Graph $C(r)$ using your graphing calculator and sketch the function below. Please label your axes to indicate the scale and window settings you used.



Window Setting

x min = 0 Y min = 0

x max = 8 Y max = 100

x scl = 1 Yscl = 10

g) For what value of r is the cost, C, the least?

If the value of r is 3.84 inches, the cost, C would be the least, \$ 19.51.

h) Find and describe a real-world application that requires one to determine the maximum or minimum of the function.

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In the real world, when people find the drug concentration in a patient's bloodstream after injection, they will determine the time at which the concentration is highest.
For example, the function is $C(t) = \frac{t}{2t^2+1}$, which C means the concentration of drug and t stands for the time (hours), the concentration will be the highest when $t \approx 0.71$ hours. Besides this, the concentration of drug decreases to 0 as time increases.